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Effects of radiation on MHD natural Convection flow from a porous vertical plate

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ABSTRACT
The present paper is devoted to Investigation on effects of radiation on natural convective flow of a viscous incompressible fluid from an infinite moving porous hot vertical plate. The non dimensional governing ordinary partial differential equations have been solved for velocity u and temperature θ by the application of Laplace transform technique. The effects of different parameters M (Hartmann number) Gr (Grashof number)r(Radiation parameter) and Pr (Prandtl number) occuring in the problem have been discussed with the help of tables and graphs. The expressions for shearing stress and heat transfer rate have been derived in this chapter.

Keywords : Viscous incompressible fluid, Natural convection, Porous, medium, radiation, MHD and Laplace Transform.

INTRODUCTION
Natural convection flow is frequently happens in nature. Recently flows of fluid through porous media are of chief interest of several research scholars on account of their applications in the field of science and technology. Investigations in flow through porous medium are solely based on Darcy's law. Now a days the developments in modern technology have intensified more interest of many investigators in the studies of heat and mass transfer in fields due to its wide applications in oil reservoir and isothermal engineering as well as other geophysical & astrophysical studies. A theoretical and experimental work on this topic can be seen in the monographs by Nield and Benjan [1]. Ingham and Pop [2], Kim and Vafai [3] have analyzed the buoyancy driven flow about a vertical plate for constant wall temperature and heat flux. Raptis and Singh [4] studied the flow past an impulsively started vertical plate in a porous medium by a finite difference method. Seigel [5] first studied transient free convection flow past a semi infinite vertical plate by an integral method. Since then several investigators have got their papers published on natural convection flow past a semi-infinite vertical plate. Soundalgekar et. al [6] have studied free convection flow past a vertical porous plate. Yomemoto et. al [7] have studied the acceleration of convection in a porous permeable medium along an arbitrary but smooth surface. Raptis [8] investigated free convection in a proous medium bounded by an infinite plate.

Raptis and Perdikis [9] studied free convection flow through a porous medium bounded by a semi-infinite vertical porous plate numerically. Sattar [10] has analyzed the same problem and obtained analytical solution bypertubation Teachnique used by Singh and Dikshit [11].

MHD natural convection from the technological point of view have great significance due to the applications in the fields of steller and planetary magneto spheres, aeronautics, chemical engineering and electronics. The effects of magnetic field on natural convection flow of electrically conducting fluids past a porous medium has been investigated by several research scholars. Ahmed [12] has studied the effects of unsteady free convection MHD flow through a porous medium bounded by an infinite vertical porous plate. Chaudhary R.C. and Arpita Jain [13] have studied the MHD heat and mass transfer diffusion flow by natural convection past a surface embedded in a porous medium.

Soundalgekar [14] determined approximate solutions for two dimensional flow of an incompressible viscous fluid past an infinite porous vertical plate with constant suction velocity normal to the plate and the difference between the temperature of the plate and the freestream is moderately large causing the free convection currents.Kim [15] investigated unsteady MHD convection flow of polar fluids past a semi-infinite vertical moving porous medium. Raptis [16] discussed mathematically the case of unsteady two dimensional natural convection heat transfer of electrically conducting incompressible viscous fluid through highly porous medium bounded by an infinite vertical porous plate. Several other researchers like Soundelgekar [17] and Singh et.al [18] have studied the effects of magnetic field on free convection flow of electrically conducting fluids past a plate.

All the aforesaid investigations are due to MHD flow and heat transfer problems only. However, the radiation influences on MHD flow and heat transfer investigations have become more essential for industrial point of view. Athigh operating temperature, the effects of radiation can be quite significant. Sparrow[19] described a parameter termed Rosseland approximation to explain radiation heat flux in his book in energy equation. Hossain and Takhar[20] studied effects of radiation on mixed convection along a vertical plate with uniform temperature. Molta et. al[21] investigated natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation / absorption. Akhtar[22] studied effect of radiation on free convection flow on sphere with isothermal surface and uniform heat flux. Ali [23] studied the radiation effect of

free convection flow on sphere with heat generation. The radiative flows of an electrically conducting fluid with high temperature in the presence of magnetic fields are encountered in electrical generation, solar power, technology, space vehicle entry, astrophysical flows, nuclear energy applications and many other industrial areas. The effects of radiation on boundary layer flow with and without a magnetic field under different condition have been studied by researchers. For example Mahmoud [24], Israel-cookey et.al [25], Hayat et.al [26]. The heat and mass transfer owing to radiation play influencive role in manufacturing industries for the design of fins, nuclear power plants, steel rolling, Gas turbines and different devices for aircrafts, Missiles satellites and space vehicles are the examples of such engineering applications.Prasad et. al [27] investigated radiation effects and mass transfer effect on unsteady MHD free convection flow past a porous vertical plate embedded in porous medium. Mohammed Ibrahim et.al[28] studied the radiation and chemical reaction effects on MHD free convective flow past a moving vertical plate.

The present problem is devoted to the investigations on effects of radiation on MHD natural convection flow of viscous incompressible fluid from a porous hot vertical infinite moving plate. The non dimensional governing partial differential equations of motion have been solved for velocity u and temperature θ by applying Laplace transformation technique. The effects of different parameters M (Hartmann Number), Gr (Grashof Number), R (Radiation Parameter) and PR (Prandtl number) occurring have been discussed with the help of tables and graphs. The expressions for shearing stress function and heat transfer rate have been derived in this section.

FORMULATION OF PROBLEM

The unsteady convective flow of MHD viscous incompressible fluid bounded by a vertical infinite moving hot porous plate under the influence of radiation and porous medium has been considered. We consider a cartesian reference frame of co-ordinates such that x* – axis is along the plate in the vertically upward direction and y* axis is in the direction normal to the plate.

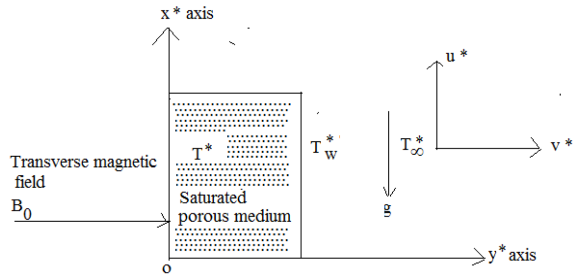

Fig – I (Geometry of Flow and Coordinate System)

A transverse magnetic field of uniform strength Bo is applied normal to porous plate. We consider some assumptions i.e. polarization effect, viscous dissipation, induced magnetic field are ignored. Under above assumptions the governing equations of flow problem are.

Equation of continuity

$$\frac{\partial u*}{\partial x*} + \frac{\partial v*}{\partial y*} = 0 \qquad (5.2.1)$$

Momentum Equation

$$\frac{\partial u*}{\partial t*} = \upsilon \frac{\partial u*}{\partial y*^2} + g\beta(T_w^* - T_\infty^*) - \frac{\sigma_0 B_0^2}{\rho} - \frac{\upsilon}{k'} u^*$$

(5.2.2)

Energy Equation

$$\frac{\partial T*}{\partial t*} = \frac{k_1}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y^*}$$

(5.2.3)

The suitable boundary conditions on the wall of plate as well as in the free stream are given by

$t^* \leq 0$, $u^*=0$, $T^*=T_\infty^*$ for every y^*

$t^*>0$, $u^*=0$, $T^*=T_w^*$ at $y^* = 0$

$u^* \to 0$, $T^* \to T_\infty^*$ as $y^* \to \infty$

(5.2.4)

Where (u^*, v^*) are the components of velocity of fluid along x^* and y^* axes respectively. T^*, T_w^*, T_∞^* are temperatures of the fluid at the plate and away from plate respectively. t^* is the time, υ kinematic co-efficient of viscosity, $\rho$the density of fluid, σ_0 electrical conductivity. C_p specific heat at constant pressure, g is the acceleration due to gravity,β co-efficient of thermal expansion, B_0 magnetic field of uniform strength, q_r is the radiative heat flux and K' is the permeabilityparameter of porous medium.

From the application of Rosseland [29] approximation of radiative flux vector q_r can be written as

$$q_r = \frac{4\sigma^*}{3k^*}\frac{\partial T^{*4}}{\partial y^*} \dots\dots\dots\dots\dots\dots\dots\dots$$

(5.2.5)

Further it is assumed that the temperature differences within the flow are sufficiently small therefore T^{*4} can be expended in a Taylor series about the free stream temperature T_∞^* so that the higher order terms are neglected.

i.e $T^{*4} = 4T_\infty^{*3}T^* - 3T_\infty^{*4} \dots\dots\dots\dots\dots\dots\dots$

(5.2.6)

Now using (5.2.5) and (5.2.6) in the energy equation (5.2.3) we obtain new energy equation in the form

$$\frac{\partial T*}{\partial t*} = K_1\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{1}{\rho C_p}\frac{4\sigma^*}{3k^*}\frac{\partial T^{*4}}{\partial y^*}$$

(5.2.7)

Where σ^* is Stephen Boltzmann constant, K_1 is thermal conductivity, K^* mean absorption coefficient of the medium.

Now in order to non-dimensionalize the equations (5.2.2) and (5.2.7) we introduce a set of non-dimensional variables defined as

$$u = \frac{u^*}{U_0}, y = \frac{y^* u_0}{\upsilon}, t = \frac{U_0^2 t^*}{\upsilon}$$

$P_r = \frac{\rho \upsilon C_p}{K_1}$ (Prandtl Number)

$G_r = \frac{\upsilon g\beta(T_w^* - T_\infty^*)}{U_0^3}$ (Grashof Number)

$R = \frac{16\sigma^* T_\infty^{*3}}{3k^* K_1}K'$ (Radiation Parameter)

$\lambda = \frac{K' U_0^2}{\upsilon^2}$ (Permeability Parameter of porous medium)

$\theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}$ (Dimension less temperature)

(5.2.8)

The law of conservation of mass i.e. equation of continuity holds good and the equations (5.2.2) and (5.2.7) are made non-dimensional in the form.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r\theta - \left(M + \frac{1}{\lambda}\right)u$$

(5.2.9)

$$(1+R)\frac{\partial^2 \theta}{\partial y^2} - P_r\frac{\partial \theta}{\partial t} = 0$$

(5.2.10)

The relevant corresponding boundary conditions as follows.

For $t \leq 0$, $u=0$, $\theta= 0$ for all y

$t>0$, $u=0$, $\theta = 1$ for $y = 0$

$u \to 0$, $\theta \to 0$ as $y \to \infty$

(5.2.11)

METHOD OF SOLUTION

Here we adopt usual Laplace transform technique in order to solve the linear partial differential equations (5.2.9) and (5.2.10) for velocity field u and temperature field θ.

If Laplace transform of the function u (y, t) is defined as$\bar{u}(y,p)$ then

$$\bar{u}(y,p) = \int_0^\infty e^{-pt}u(y,t)dt \qquad (p>0)$$

(5.3.1)

then equation (5.2.9) and (5.2.10) are transformed into

$$p\bar{u} = \frac{d^2\bar{u}}{dy^2} + G_r\bar{\theta} - \left(M + \frac{1}{\lambda}\right)\bar{u}$$

(5.3.2)

and $(1+R)\frac{d^2\bar{\theta}}{dy^2} - P_r p\bar{\theta} = 0$

(5.3.3)

The boundary conditions are written after transformation as

For $t \leq 0$, $\bar{u}= 0$, $\bar{\theta} = 0$ for all y
$t > 0$, $\bar{u}= 0$, $\bar{\theta} = \frac{1}{p}$ for y = 0
$\bar{u} \to 0$, $\bar{\theta} \to 0$ as $y \to \infty$

(5.3.4)

Here we consider equation (5.3.3) to find its solution for $\bar{\theta}$

i.e. $\frac{d^2\theta}{dy^2} - \frac{P_r}{1+R} p\bar{\theta} = 0$

Taking $\frac{d}{dy} = D$, the above equation is written as

$\left(D^2 - \frac{P_r}{1+R}p\right)\bar{\theta} = 0$

i.e. Auxiliary equation is

$$D^2 - \frac{P_r}{1+R}p = 0$$

i.e. D= $\pm\sqrt{\frac{P_r}{1+R}p}$

Hence

$$\bar{\theta} = Aexp\left(\sqrt{\frac{P_r}{1+R}p}.y\right) + Bexp\left(-\sqrt{\frac{P_r}{1+R}p}.y\right)$$

(5.3.5)

From the boundary condition (5.3.4), we find from equation (5.3.5)

$$A + B = \frac{1}{p}$$

and A = 0, $B = \frac{1}{p}$

$$\therefore \bar{\theta} = \frac{1}{p} exp\sqrt{\frac{P_r}{1+R}}py$$

(5.3.6)

Taking inverse Laplace transform of (5.3.6) we get

$$\theta(y,t) = erfc\left[\frac{y}{2}\sqrt{\frac{P_r}{(1+R)t}}\right]$$

(5.3.7)

Also substituting (5.3.6) into (5.3.2) and following Vineet Kumar Singh et.al. [30] we find the solution for u(y,t) as

$$u(y,t) = \frac{1}{2}\left(1 - \frac{G_r}{M+\frac{1}{\lambda}}\right) \times \left[\exp\left\{-\left(\sqrt{M+\frac{1}{\lambda}}\right)y\right\} erfc\left(\frac{y-2\sqrt{M+\frac{1}{\lambda}t}}{2\sqrt{t}}\right) + \exp\left\{\left(\sqrt{M+\frac{1}{\lambda}}\right)y\right\} erfc\left(\frac{y+2\sqrt{M+\frac{1}{\lambda}}}{2\sqrt{t}}\right)\right] + \frac{G_r}{M+\frac{1}{\lambda}} erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{(1+R)t}}\right) + \frac{G_r}{M+\frac{1}{\lambda}}\frac{y}{2\sqrt{\pi}}\frac{(\sqrt{2}-1)t^{\frac{1}{2}}}{2} \times \left[\exp\left\{-\left(\frac{y^2}{4t} + \left(M+\frac{1}{\lambda}\right)t - \sqrt{\frac{P_r}{1+R}} exp\left(-\frac{y^2}{4t}\frac{P_r}{1+R}\right)\right)\right\}\right]$$

(5.3.8)

The expressions for shear stress function i.e. velocity gradient, and surface heat temperature i.e. temperature gradient take the form.

$$\left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{1}{2}\left(1 - \frac{G_r}{M+\frac{1}{\lambda}}\right) \times \left[\begin{array}{c} -\left(\sqrt{M+\frac{1}{\lambda}}\right) erfc\left\{-\left(M+\frac{1}{\lambda}\right)\sqrt{t}\right\} \\ \left(M+\frac{1}{\lambda}\right) erfc\left\{\left(M+\frac{1}{\lambda}\right)\sqrt{t}\right\}\frac{2}{\sqrt{t\pi}}\exp\left\{-\left(M+\frac{1}{\lambda}\right)t\right\} \end{array}\right] - Gr\lambda\frac{1}{\sqrt{\pi}}\left[\sqrt{\frac{P_r}{(1+R)t}}\right] + \frac{G_r}{M+\frac{1}{\lambda}}\frac{y}{4\sqrt{\pi}}\frac{(\sqrt{2}-1)t^{-\frac{1}{2}}}{2}\left[\exp\left\{-\frac{1}{\lambda}t\right\} - \sqrt{\frac{P_r}{(1+R)}}\right]$$

(5.3.9)

and $\left.\frac{\partial\theta}{\partial y}\right)_{y=0} = -\frac{1}{\sqrt{\pi}}\left(\frac{P_r}{(1+R)t}\right)$

(5.3.10)

RESULTS AND DISCUSSION

In this section we have found outthe exact solutions of velocity and temperature distributions in the investigation of magneto hydrodynamic natural convection incompressible viscous fluid flow past a vertical surface with porous medium along with radiation. The effect of M and G_r on the velocity distribution u has been analyzed in figures (1) and (2). Further, the influence of P_r and R on temperature distribution has been exhibited in figures (3) and (4)

It is noticed from figures (1) and (2) the graphs of velocity begin to decrease as values of y increase. It is also observed that all the graphs increase with the increasing M and decrease while G_r increases.

From figures (3) and (4) we study that all the graphs of temperature θ starts to decrease as values of y increase. It is also observed the graphs decrease with the increase in P_r whereas the graphs increase with the increase in R.

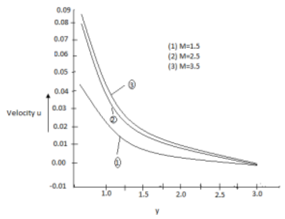

Fig.1 Velocity distribution u for $G_r = 2$, $P_r = 0.71$, R = 1 and t = 0.2 for various value of M.

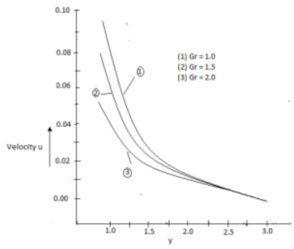

Fig.2 Velocity distribution u for M = 1.5 , $P_r = 0.71$, R = 1 and t = 0.2 for various value of Grashof numbers G_r.

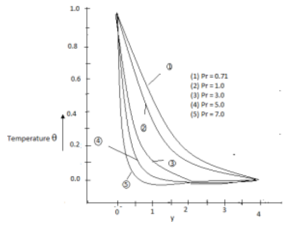

Fig.3 Temperature distribution θ for R = 1.0 , t= 0.2, for various value of P_r.

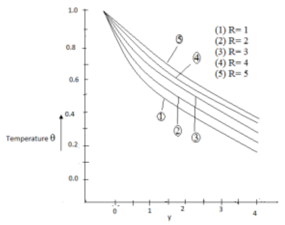

Fig.4 Temperature distribution θ for $P_r = 1.0$, t= 0.2, for various value of R.

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